# A STUDY OF PROPELLANTS AND DELIVERED SPECIFIC IMPULSE IN MODEL ROCKET ENGINES

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# PREFACE

The author would like to thank the Aerojet Strategic Propulsion Company of Sacramento, California for their assistance in the preparation of this paper. Mr. J. D. Mockenhaupt of the Aerophysics Section has been of great assistance in offering comments and technical advice.

# I. INTRODUCTION

This paper was written at the request of Pat Miller, the President of the National Association of Rocketry. Mr. Miller was specifically interested in the "upper limits of a 62.5 gram motor using class 1.3 propellant and the lower limits of an F motor (size, weight)." Although a similar study was done previously and presented at NARAM-24, it was somewhat flawed in its approach to the problem of what the limits of rocket motor technology were, due to some flawed assumptions on the practicality of applying some current full scale rocket technology to model rocket engines.

The objective of this report is to apply some sound engineering calculations to the problem of predicting what the future limits of chemical propulsion technology are.

The chemical ingredients which affect the delivered performance of a propellant will be considered in order. These ingredients are binders, oxidizers,and metal additives. The physical factors which affect motor performance will also be examined. These factors are propellant composition, operating pressure, and loss mechanisms.

In the section on inerts, the limits will be estimated by comparing mass fractions of solid propellant motors of varying sizes to current model rocket engines.

## II. PROPELLANTS

The propellants which power current model rocket engines are of two generic types: black powder and unmetalized composites. We shall not deal further with black powder as its future is fairly certain. Composite propellants consist of an elastomeric binder which also serves as a fuel, a ground crystaline oxidizer, and in the case of "professional" motors, a metal fuel. Various other minor additives such as cure agents, burning rate catalysts, and antioxidants are also added but these additives are present in such small quantity that they do not affect the performance (Isp) of the propellant. Each of these major constituents will be discussed in turn.

### A. BINDERS

There are many elastomeric binders which have been used in rocket propellants in the past. These binders have included polyesters, polysulfides, polybutadiene acrylonitrile (PBAN), carboxy terminated polybutadiene (CTPB) and hydroxy terminated polybutadiene (HTPB). With the advent of HTPB as a propellant binder in the early 1970's the use of the other types has steadily declined. The reason is simple: there is no other current binder system which has better processability or higher energy.

All of the new generation of strategic missiles utilize HTPB as a binder (at least in the lower stages). This includes C-4, MX stages I and II, IUS, and others. It is also used in a good many tactical motors. Only where the propellant is locked in by specification or where concerns such as manrating (i.e. Space Shuttle) take precedence, is the use of HTPB restricted. (This is not out of concern for the safety of the binder but rather is a reflection of the conservatism in the Air Force and NASA).

There are currently no other binder systems on the horizon which are likely to replace HTPB. Some work has been going on recently, conducted by the Air Force Rocket Propulsion Laboratory (AFRPL) on glicidyl azide polymers (GAP) which promise to be more energetic than HTPB. This type of binder is designed to be compatible with the explosive plasticizer nitroglycerin (NG). The class of NG/GAP propellants is almost certain to be a class 1.1 high energy propellant which because of its mass detonation hazard could not be classified as DOT class C in any quantity.

Another possibility for increasing the energy of propellant binders is to use a nitroplasticizer as a constituent of the binder. This approach which was used successfully in Polaris (nitroplasticized polyurethane) offers only a very limited boost in performance. The clincher for model rocket applications is the fact that there are perhaps only one or two manufacturers of the stuff in the free world and because of the limited quantity produced, the cost is high and the availibility is limited.

Regarding high energy additives it is perhaps worth mentioning here that there are some other additives, binders, and binder constituents of modern high energy propellants which offer real boosts in delivered Isp. A few of these include HMX, RDX, HNS, DEGDN, TMETN, and the FEFO, SYEP, SYFO and the PEG/NG family of binders. The drawbacks of these compounds are varied, ranging from explosive hazard, to cost, toxicity, and availibility. The real issue affecting model rocket propellants is that the use of any one of these in sufficient quantity to boost performance will result in a class 1.1 propellant which can not be classified as DOT class C in any quantity.

The bottom line is this: since model rocket engines already use HTPB as a binder, no great increases in Isp due to binder systems will occur in the near future. To quantify this statement is is prudent to observe that in the past twenty years the Isp of class 1.3 propellant in the rocket industry as a whole increased by only 5%. In the past ten years the Isp of propellants has increased by less than 1%. It was for this reason and similar reasons related to hardware development, that the AFRPL program on the Avanced Technology Upper Stage (ATUS) was cancelled. The feeling could be summed up in a single sentence which says: "Why spend millions of dollars developing new technologies when the payoff is only a 5% boost in performance?" We are currently so far up the power curve with chemical propulsion technology that the curve is nearly flat. So to quantify the opening sentence of this paragraph I would say that the following is probable: 10 years -- 1% boost in current Isp, 20 years -- 5% boost in current Isp due to binder alone. This raises the question as to what is the current available Isp. This question will be answered in section V.

## B. OXIDIZERS

The current oxidizer of choice in the rocket industry is ammonium perchlorate (AP). Unfortunately (for the rocket industry) it has been the oxidizer of choice for the past thirty years. Other common solid oxidizers exist. These include ammonium nitrate (AN), potassium nitrate, potassium perchlorate (KP), sodium nitrate and lithium perchlorate. All of these oxidizers have much lower Isp than AP except for lithium perchlorate which is only very slightly lower. The problem with lithium perchlorate is that it is hygroscopic and expensive.

Some years ago the Air Force spent a great deal of money in an attempt to improve the performance of oxidizers. They zeroed in on nitronium perchlorate (NO2ClO4), but they were never able to solve three nasty problems: aging, moisture sensitivity and detonation. Nitronium perchlorate is simply the anhydride of nitric and perchloric acids and as such, if the compound comes in contact with any moisture -- even atmospheric humidity, the results are rather foul and corrosive.

There is a class of heavy metal oxidizers such as lead nitrate which is used in certain specialized applications such as ejection seat motors where the power per unit volume is critical. (This is called density-Isp, $\rho$-Isp). Because model rocket motors are limited by propellant weight rather than by volume these oxidizers are of no concern here, for although density-Isp is high, Isp itself is much lower than with AP. This is to say nothing of the problems of blowing all kinds of heavy metal oxides all over the place.

Getting away from the classical binder/oxidizer approach there are some systems which offer higher Isp. One such system is the NG/PEG/HMX/AP system similar to the ones used on some advanced upper stage motors. The problem is that all of these systems are class 1.1 because of their NG or HMX or whatever other high explosives are used to increase the energy of the mixture.

The future of oxidizers is bleaker than that of binders. In the past thirty years there has been no change in the energy available in class 1.3 oxidizers. None is forseen in the next twenty years. The net effect on model rocket propellants: 0% change in Isp due to improved oxidizers in the forseeable future.

## C. METAL ADDITIVES

The big jump in class 1.3 propellant Isp came about 25 years ago with the addition of aluminum powder to a composite propellant. This discovery was made during the development of the Polaris missile. During this era just about every element in the periodic table was investigated for possible use in a propellant formulation. Several possible additives were identified which looked promising, these included aluminum, boron, beryllium, magnesium, and zirconium along with a few others.

Beryllium is by far the best metal additive from a performance standpoint, unfortunatley human beings have the unfortunate habit of dying when exposed to beryllium or beryllium oxides. This limits the locations where propellants containing beryllium can be mixed (like some place in Nevada) and fired (like some place in deep space).

Aluminum turned out to be the best compromise from the standpoint of performance, cost and environmental effects. It boosts theoretical Isp by up to 20% and has found its way into just about every rocket motor built in the last ten years except where the brilliant white plume and exhaust trail of aluminum oxide are a tactical disadvantage. (Such as anti-tank weapons and air to air missiles).

Even with all its advantages aluminum still has not been widely used in model rocket motors to increase performance. The reason which is given is as follows: Aluminum combustion in a rocket engine is a slow process. In large motors this is not a problem because the residence time of the aluminum particles in the chamber is sufficiently long to burn nearly all the aluminum before it exits from the chamber through the nozzle. As the size of the motor decreases the residence time drops rapidly until the size of a model rocket motor is reached and supposedly so little time is available for burning aluminum in the chamber that most of it exits from the nozzle unburned.

This line of reasoning is probably valid for small motors with high aluminum contents (15-20%), however there is evidence in the form of motor firings conducted by the Chemical Systems Division of United Technology Corporation which indicate that the combustion efficiency of small motors containing about 5% aluminum is very high. Due to a lack of theoretical and experimental data in this area it will be assumed for the purposes of this paper that there are no performance losses due to incomplete combustion (i.e. $\eta_{ce} = 1.00$).

Aluminum remains at this point a viable candidate for increasing the delivered Isp of a model rocket propellant. It will be considered further in sections III-B and III-C.

Boron is has a vapor phase combustion and thus does not have the combustion efficiency problems which plague aluminum. However boron oxide condenses very rapidly on cold chamber walls and nozzle surfaces severely affecting performance. Experimental data has shown that this effect does not fade until about 4-5 seconds into a firing when the nozzle surfaces become hot enough to boil off all the boron oxide. In addition boron is expensive and this fact alone would probably preclude its use in model rocket propellants.

Zirconium like boron is not likely to be used in a model rocket engine for simple economic reasons. The current price of zirconium is about $150.00 per pound.

A metal additive to model rocket propellants which has been speculated about is magnesium. Magnesium would allow a 7.7% boost in theoretical Isp over unmetalized propellants (at 2000 psia). It is not known how magnesium would behave in a small rocket engine. Based on its similarity to aluminum and boron it is expected that the addition of magnesium to a propellant would tend to decrease rather than increase the delivered Isp of the propellant. There is a small amount of evidence from the melting and boiling points of magnesium oxides that this would not happen. The only real way to find out is to fabricate and test a motor containing magnesium.

Magnesium like aluminum remains a viable candidate for addition to a model rocket propellant and is considered further in the next section.

Before predicting what effect metal additives will have on model rocket propellants in the future, the real-world losses due to two phase flow must be taken into account. This is treated in section III-C. With these losses factored in, the predicted increase in Isp due to metal additives is as follows: 1-10 years -- 2.3% increase over current Isp, 20 years -- 4.6% increase over current Isp.

## III. PHYSICAL FACTORS

### A. PROPELLANT COMPOSITION

Figure 1. shows the effect of increasing the quantity of ammonium perchlorate in an unmetalized propellant. It is observed that the theoretical Isp increases up to about 90% AP by weight and then starts decreasing. The effect of pressure is also shown in the three curves at 500, 1000, and 2000 psia. One could deduce that the optimum model rocket motor would contain about 90% by weight AP, and from a performance standpoint this would be correct. Such a propellant is not castable however. The limit for casting propellants is about 84 to 86% solids by weight. Even at this level of loading, careful attention must be paid to the oxidizer particle size distribution otherwise the propellant becomes a paste. Paste propellants have been used in model rocket motors in the past. They must be hand tamped into place. This castability limit is not one of performance, but of cost. Use of a paste propellant only limits the likelyhood of economic viability, but is not a show-stopper as far as performance is concerned.

### B. OPERATING PRESSURE

Figure 2. shows the effect of increasing the motor operating pressure on theoretical Isp. The curves represent % by weight AP ranging from 80% to 95%. Current composite rocket motors operate at about 500 psia and at 84% AP. If the pressure of this type of motor were increased to 2000 psia the theoretical Isp would rise from 227.0 lb-sec/lbm to 253.1 lb-sec/lbm, an increase of 11.5%. The motor case would have to be strenghtened and the nozzle expansion ratio would have to be increased to take advantage of this gain. This area of higher pressure motors is the largest area available to the model rocket manufacturer wanting to increase the delivered Isp of a rocket motor. It entails certain risks such as the danger of a high pressure case burst or nozzle ejection, and certain design problems such as ensuring stable ignition and propellant combustion. None of these obstacles are insurmountable.

### C. LOSS MECHANISMS

As in all real-world heat engines there are certain losses which occur and can not be avoided. Much like the Carnot cycle efficiency, the theoretical Isp can be approached, but never equalled. In a rocket engine the losses are divided into five types: combustion efficiency loss, boundary layer loss, divergence loss, kinetic loss, and two-phase flow loss. Thus the delivered Isp of the propellant in a rocket motor can be written as:

$$\text{Isp(delivered)} = \eta_{ce} * \eta_{bl} * \eta_{div} * \eta_{kin} * \eta_{tpf} * \text{Isp(theoretical)} \quad \text{[A]}$$

[A] this equation assumes a nozzle discharge coefficient of 1.00 and a perfectly expanded nozzle (Pe = 14.7 psia).

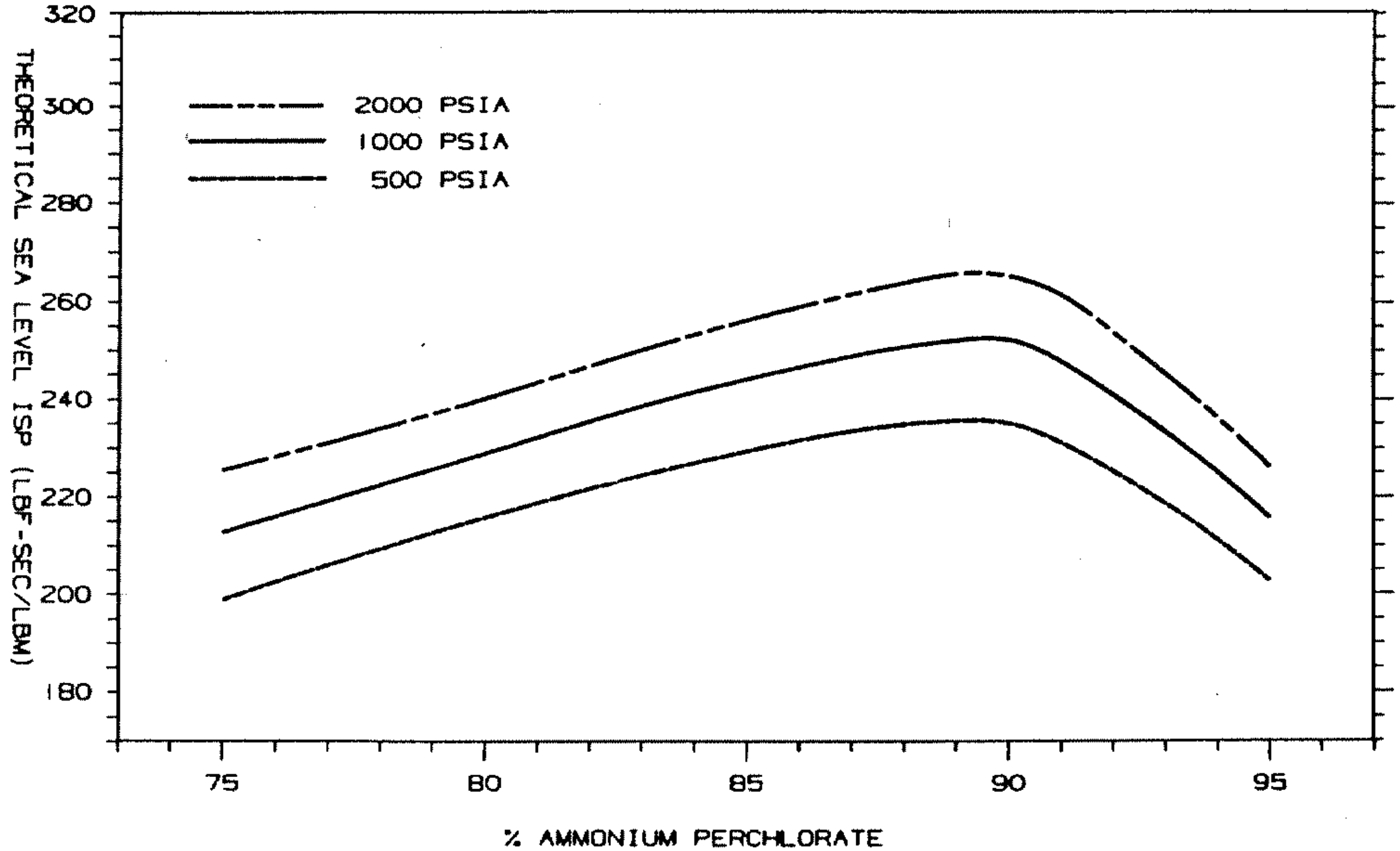
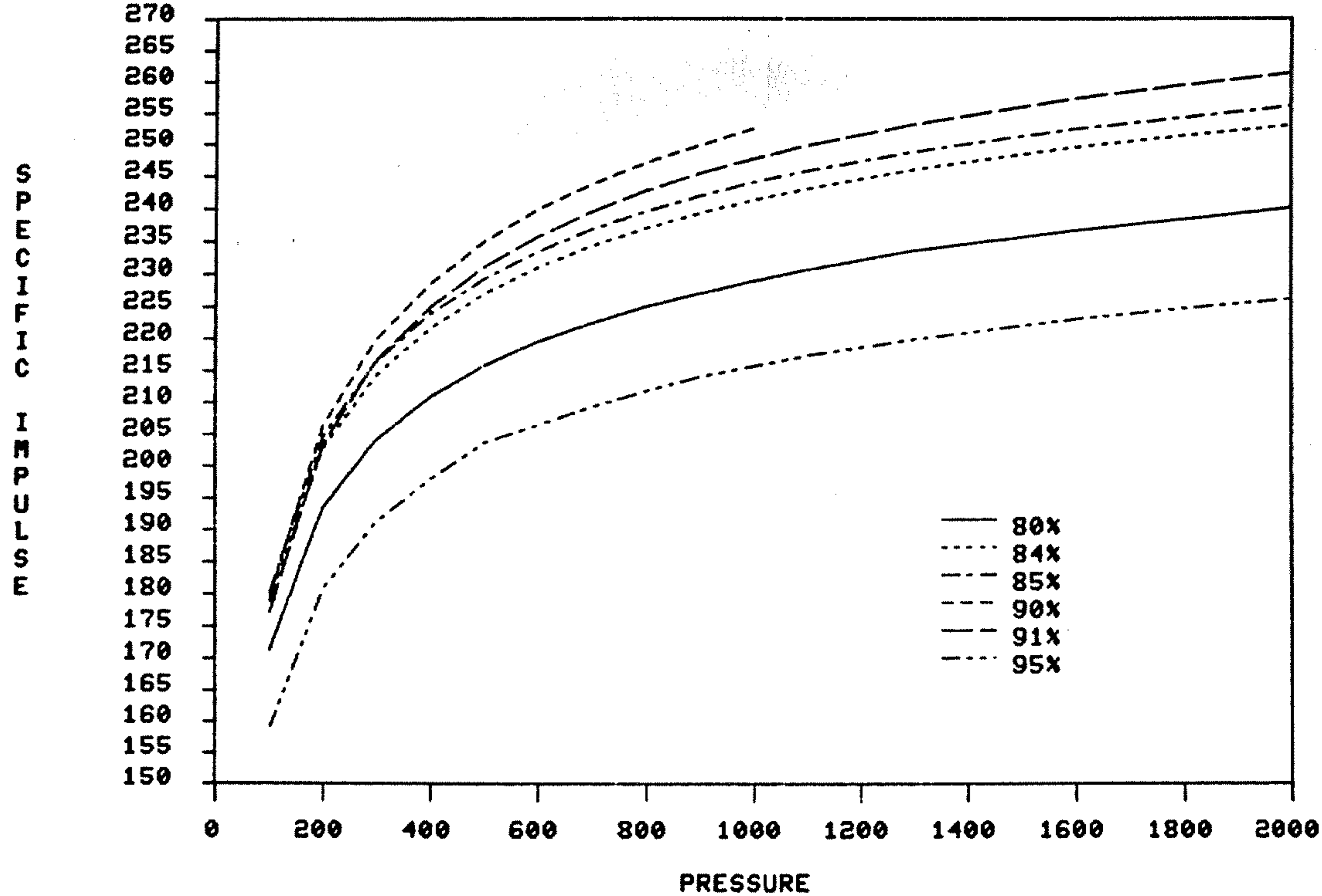



FIG. 1 -- THEORETICAL ISP VS OXIDIZER CONTENT
R-45 BINDER (HTPB)

## UNMETALIZED PROPELLANT
## ISP VS PRESSURE



For the purposes of this report the combustion efficiency was assumed to be 1.00. This avoids the problem of predicting combustion efficiency which has no solid theoretical basis for small motors anyway. It will not affect the magnitude of the predictions in any case as will be shown.

Boundary layer losses are due to boundary layer effects as the name implies. The fluid in a nozzle has zero velocity at the wall and has the free-stream velocity in the center of the flow. The velocity gradient between the free-stream velocity and the wall is called the boundary layer. In a rocket nozzle the boundary layer is usually turbulent and results in a considerable drag force on the gas flowing out the nozzle. The boundary layer loss for a typical composite F-engine such as the Aerotech F-44 is calculated to be 2.12% ($\eta$ bl = .9788). [B]

Divergence loss is a function of the half angle of the rocket nozzle. Almost all composite rocket motors produced to date have a straight conical exit cone with a half angle between 15 and 20 degrees. Because the gas exiting the nozzle has a significant radial component equal to the sine of the half angle multiplied by the exit gas velocity, there is a performance loss. This loss is due to the fact that the radial component of the gas velocity produces no net forward thrust. The magnitude of the divergence loss for a motor such as the F-44 is 1.70% ($\eta$ div = .9830).

Kinetic loss is due to the finite rate of chemical reactions in the gas stream as it flows down the nozzle. These reactions are driven by the rapidly decreasing temperature and pressure of the gas stream. The lower pressure tends to favor lower molecular weight species while the decreasing temperature favors higher molecular weight species. At one extreme the maximum energy available from the gas stream results from a continuously shifting equilibrium composition (reaction rate = infinite). The Isp calculated for this assumption is called Isp theoretical or Isp shifting. At the other extreme the assumption is made that the equilibrium composition of the gas is "frozen" or fixed at the chamber and there is no change in the composition as the gas flows down the nozzle (reaction rate = zero). The Isp for this assumption is called Isp frozen. The kinetic loss is generally very small in magnitude. For the F-44 it is calculated at 0.14% for a propellant with no metal ($\eta$ kin = .9986). The kinetic loss increases slightly for increasing metal content, for example a propellant with 20% magnesium by weight has a kinetic loss of 1.04% ($\eta$ kin = .9896). This increase is due to the changing gas composition (more fuel rich) with increased metal content. [C]

[B] See appendix A.

[C] For a treatment of equilibrium thermodynamics refer to Sonntag & Van Wylen, Intro. to Thermodynamics, ch. 14, J. Wiley & Sons Inc. 1971.

Two-phase flow loss is caused by the exhaust products in the nozzle having two components or phases, a gaseous phase and a liquid or solid phase consisting of condensed metal oxide particles. Obviously, for a propellant with no metal the two phase flow loss is 0%. The loss is caused by the particles because the particles of oxide ($MgO$ or $Al_2O_3$) do not expand like the exhaust gas and thus do no work. In addition the particles must be accelerated by the gas stream and the drag which results from this acceleration represents energy lost to the gas flow. one might be tempted to hazard a guess that the two phase flow loss should be small because the particles in the exhaust stream are very small and easily accelerated by the gas and that their momentum would not be far different from an equivalent weight of gas. This assumption would be reasonable except for the fact that the particle acceleration near the nozzle throat is in excess of 30 million feet per second squared and thus the drag loss is quite substantial.

Exact predicition of two phase flow loss requires a knowledge of both particle size and drag coefficients. This turns out to be a formidable problem for which no solid theoretical solution exists. Several semi-empirical methods have been developed but they would have to be extrapolated far beyond their intended range in order to be applied to model rocket motors. Fortunately it is possible to establish upper and lower bounds on two phase flow loss by fixing two boundary conditions of the basic differential equation governing two phase flow. These boundary conditions are:

1. Particle drag coefficient can be zero ($V_p=0$) or infinite ($V_p=V_{gas}$).
2. Particle heat transfer coefficient can be zero ($T_p=\text{constant}$) or infinite ($T_p=T_{gas}$).

It turns out that the lower bound on two phase flow loss is for $V_p=V_{gas}$ and $T_p=T_{gas}$. This gives the highest $\eta_{tp}$ which is possible. $\eta_{tp}$ in this case is defined as follows:

$$\eta_{TP} = \frac{V_e}{V_{eg}}$$

Where $V_e$ is the exit velocity of the total exhaust stream
and $V_{eg}$ is the exit velocity of the exhaust gas under the same contitions. [D]

For the above boundary conditions the two phase flow loss for an aluminum propellant with 20% Al can be as high as 12.84% ($\eta_{tp}$ = .8716), and for a magnesium propellant with 20% Mg as high as 10.54% ($\eta_{tp}$ = .8946).

[D] For a complete explanation of boundary conditions for two phase flow see Hill & Peterson,Mechanics and Thermodynamics of Propulsion, Addison Wesley, 1970.

Figure 3 and figure 4 combine all the above mentioned losses for both an aluminum propellant and magnesium propellant and include the effects of variation in metal content in a motor such a the F-44. As can be seen from figure 3 the theoretical Isp rises with increasing magnesium content (top curve) while the delivered Isp (bottom curve) decreases with increasing metal content. Thus the addition of magnesium to a model rocket motor has no effect other than to decrease the Isp. Figure 4 shows that for an aluminum propellant the delivered Isp peaks at about 5% aluminum by weight. The low height of the peak in the bottom curve in figure 4 shows that the addition of aluminum to a model rocket motor has only a slight positive effect on delivered Isp. The limit for an aluminized propellant is a 2.3% boost in delivered Isp with 5% aluminum at 2000 psi. This is then the near term (1-10 yr) boost is Isp which was cited in section II-C. Doubling this gives an estimate of 4.6% for 20 years down the road.

## IV. THEORETICAL LIMITS

The data from figure 4 (upper curve) has been incorporated into table 1 to show how the theoretical propellant Isp limit affects the performance of model rocket engines. A few comments about table 1 are in order: this table represents the maximum Isp, minimum weight and minimum total impulse that is available from current chemical propellant technology, no losses are assumed. Current model rocket engines operate in the range of 500 to 700 psi so that 1000 psi engines are to be considered within the realm of current technology. 2000 psi engines should be considered as a near term possibility. The bottom line of table 1 indicates that a full G-engine (160 N-sec) is theoreticaly possible although it is unlikely that this will ever be acheived.

| Chamber Pressure | Maximum Isp Theoretical | Propellant Weight 80 N-sec Motor | Total Impulse 62.5 g of Propellant |
|---|---|---|---|
| **No Metal** | | | |
| 500 psia | 2.304 N-sec/g | 34.72 g | 144.0 N-sec |
| 1000 psia | 2.473 N-sec/g | 32.35 g | 154.6 N-sec |
| 2000 psia | 2.605 N-sec/g | 30.71 g | 162.8 N-sec |
| **15% Aluminum** | | | |
| 500 psia | 2.385 N-sec/g | 33.54 g | 149.0 N-sec |
| 1000 psia | 2.557 N-sec/g | 31.28 g | 169.8 N-sec |
| 2000 psia | 2.700 N-sec/g | 29.63 g | 168.7 N-sec |

TABLE 1.

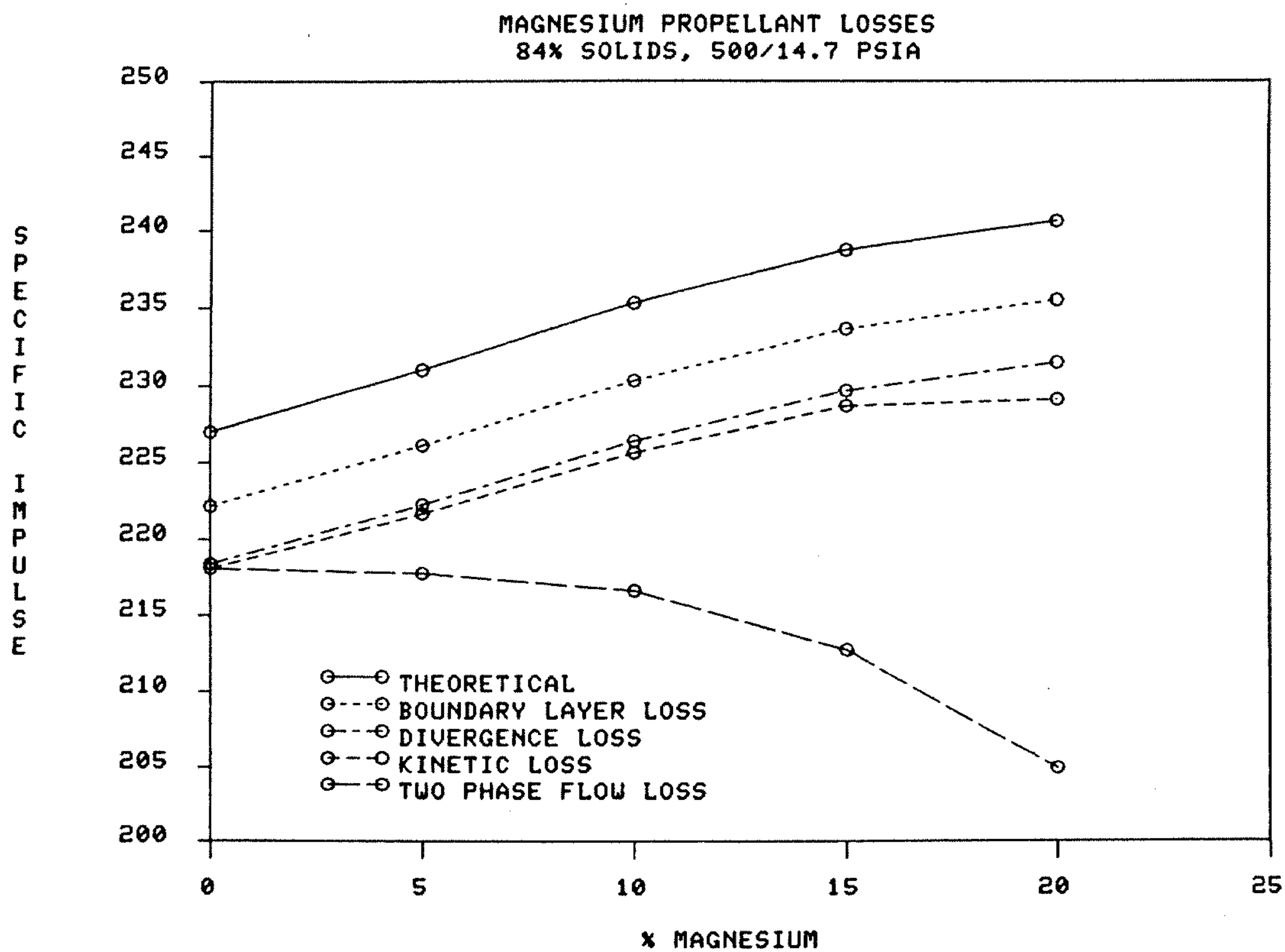

MAGNESIUM PROPELLANT LOSSES
84% SOLIDS, 500/14.7 PSIA
SPECIFIC IMPULSE
250
245
240
235
230
225
220
215
210
205
200
0
5
10
15
20
25
% MAGNESIUM
THEORETICAL
BOUNDARY LAYER LOSS
DIVERGENCE LOSS
KINETIC LOSS
TWO PHASE FLOW LOSS

FIGURE 3

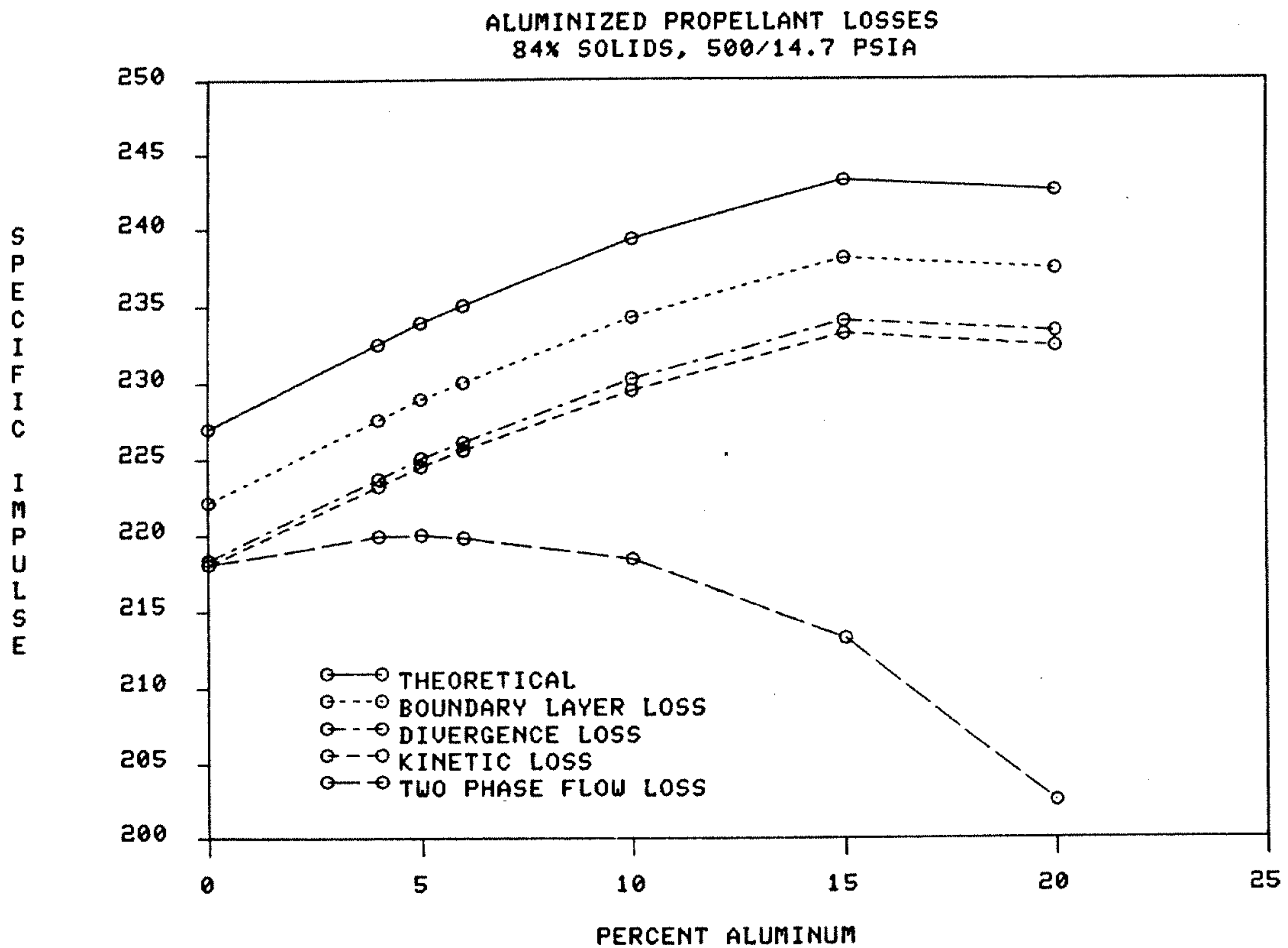

ALUMINIZED PROPELLANT LOSSES
84% SOLIDS, 500/14.7 PSIA
SPECIFIC IMPULSE
250
245
240
235
230
225
220
215
210
205
200
0
5
10
15
20
25
PERCENT ALUMINUM
THEORETICAL
BOUNDARY LAYER LOSS
DIVERGENCE LOSS
KINETIC LOSS
TWO PHASE FLOW LOSS

FIGURE 4

## V. ACTUAL LIMITS

Table 2. shows what the effects are of multiplying in the efficiency factors which were calculated in section III. These figures should be used to determine what the current limits of the technology are. As can be seen from table 2 it is not possible to get a full power G-engine with current propellant technology.

| Chamber Pressure | Maximum Isp Delivered | Propellant Weight 80 N-sec Motor | Total Impulse 62.5 g of Propellant |
|---|---|---|---|
| | No Metal | | |
| 500 psia | 2.139 N-sec/g | 37.41 g | 133.7 N-sec |
| 1000 psia | 2.245 N-sec/g | 35.63 g | 140.3 N-sec |
| 2000 psia | 2.303 N-sec/g | 34.74 g | 143.9 N-sec |
| | 5% Aluminum | | |
| 500 psia | 2.158 N-sec/g | 37.08 g | 134.9 N-sec |
| 1000 psia | 2.282 N-sec/g | 35.06 g | 142.6 N-sec |
| 2000 psia | 2.355 N-sec/g | 33.97 g | 147.2 N-sec |

TABLE 2.

## VI. FUTURE LIMITS

By multiplying the estimated future increases in Isp by the current delivered total impulse for a motor with 62.5 grams of propellant an estimate of the future limit of chemical propellant technology will be obtained. If line 2 in table 2 represents current technology then in the next 1 to 10 years the expected total impulse for 62.5 grams of propellant is 148.7 N-sec. In the next 20 years this estimate rises to 158.1 N-sec. These estimates include all losses, and all increases due to binder, oxidizer, metal additives and pressure increases. It thus appears unlikely that a full 160 N-sec G-engine using 62.5 grams of propellant will appear in the near future.

# LOSS EQUATIONS

$$\eta_{BL} = 1 - C_1 \frac{P_o^{.8}}{D_t^{.2}} \left[1 + 2e^{\left(-C_2 P^{.8}/D_t^{.2}\right)}\right] \left[1 + .016(\epsilon - 9)\right] / 100$$

$C_1 = .00365$
$C_2 = .000937$

$P_o$ = PRESSURE, PSI
$D_t$ = THROAT DIAMETER, IN
$t$ = TIME, SEC
$\epsilon$ = EXPANSION RATIO

$$\eta_{DIV} = (1 + \cos \alpha)/2$$

$\alpha$ = HALF ANGLE OF EXIT CONE

$$\eta_{KIN} = 1 - \frac{1}{3}\left[\frac{I_{SP\,TH} - I_{SP\,FROZEN}}{I_{SP\,TH}}\right] C$$

$C = 1$ FOR $P_o \leq 200$ PSIA

$C = 200/p$ FOR $p > 200$ PSIA

$I_{SP\,TH}$ = THEORETICAL (SHIFTING EQUILIBRIUM) ISP
$I_{SP\,FROZEN}$ = FROZEN (FIXED EQUILIBRIUM) $I_{SP}$

$$\eta_{TP} = \frac{V_e}{V_{eg}}$$
(MAXIMUM)

$$V_e = \sqrt{2\left[\chi C_s + (1-\chi) C_p\right] T_o \left[1 - \left(P_e/P_o\right)^N\right]}$$

$$N = \frac{R}{\left[\chi/(1-\chi)\right] C_s + C_p}$$

$$V_{eg} = \sqrt{2 C_p T_o \left[1 - \left(P_e/P_o\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$\chi$ = WEIGHT FRACTION OF CONDENSED SPECIES IN EXHAUST
$C_s$ = SPECIFIC HEAT OF CONDENSED SPECIES IN EXHAUST FT-LBF/LBM °R
$C_p$ = SPECIFIC HEAT OF EXHAUST GASSES FT-LBF/LBM °R
$T_o$ = CHAMBER STAGNATION TEMPERATURE °R
$P_e$ = EXIT PRESSURE LBF/IN$^2$
$P_o$ = CHAMBER (STAGNATION) PRESSURE LBF/IN$^2$
$\gamma$ = RATIO OF SPECIFIC HEATS OF EXHAUST GASSES $C_p/C_v$

# UNMETALIZED PROPELLANT $I_{SP}$
# VARIATION WITH OXIDIZER CONTENT

## SHIFTING EQUILIBRIUM (MAXIMUM THEORETICAL)

| % AP | % R-45 | 500 PSI | 1000 PSI | 2000 PSI |
|---|---|---|---|---|
| 95% | 5% | 202.7 | 215.7 | 226.2 |
| 92% | 8% | 225.3 | 240.9 | 253.8 |
| 91% | 9% | 231.1 | 247.7 | 261.4 |
| 90% | 10% | 235.0 | 252.2 | — |
| 89% | 11% | 235.5 | 252.0 | 265.6 |
| 88% | 12% | 234.7 | 250.6 | 263.7 |
| 87% | 13% | 233.4 | 248.7 | 261.4 |
| 86% | 14% | 231.5 | 246.4 | 258.8 |
| 85% | 15% | 229.3 | 244.0 | 256.1 |
| 84% | 16% | 227.0 | 241.3 | 253.1 |
| 80% | 20% | 215.8 | 228.9 | 240.1 |
| 75% | 25% | 198.9 | 212.8 | 225.6 |

$\frac{LB_f\text{-SEC}}{LB_m}$

## FROZEN FLOW

| % AP | % R-45 | 500 PSI | 1000 PSI | 2000 PSI |
|---|---|---|---|---|
| 95% | 5% | 199.8 | 212.7 | 223.4 |
| 92% | 8% | 217.7 | 233.0 | 245.9 |
| 91% | 9% | 221.7 | 237.4 | 250.9 |
| 90% | 10% | 224.6 | 240.7 | 254.9 |
| 89% | 11% | 226.6 | 242.8 | 256.6 |
| 88% | 12% | 227.6 | 243.7 | 257.2 |
| 87% | 13% | 228.0 | 243.6 | 256.7 |
| 86% | 14% | 227.5 | 242.6 | 255.2 |
| 85% | 15% | 226.3 | 241.0 | 253.2 |
| 84% | 16% | 224.6 | 238.9 | 250.6 |
| 80% | 20% | 214.4 | 227.1 | 237.3 |
| 75% | 25% | 196.0 | 206.8 | 215.5 |

$\frac{LB_f\text{-SEC}}{LB_m}$

FROZEN FLOW IS AT A LOWER EXPANSION RATIO BUT AT $P_e$ = 14.7 PSIA

UNMETALIZED PROPELLANT

$I_{SP}$ VS PRESSURE

SHIFTING EQUILIBRIUM

APPENDIX C

DATA FOR FIG. 2

| | % AP | | | | | |
|---|---|---|---|---|---|---|
| $P_c$ PSIA | 80% | 84% | 85% | 90% | 91% | 95% |
| 100 | 171.3 | 178.8 | 180.2 | 179.5 | 177.2 | 159.1 |
| 200 | 193.6 | 202.9 | 204.7 | 206.4 | 203.5 | 180.9 |
| 300 | 204.2 | 214.4 | 216.5 | 220.0 | 216.7 | 191.4 |
| 400 | 210.9 | 221.7 | 224.0 | 228.6 | 225.0 | 198.0 |
| 500 | 215.8 | 227.0 | 229.3 | 235.0 | 231.1 | 203.7 |
| 600 | 219.5 | 231.0 | 233.4 | 239.9 | 235.7 | 206.4 |
| 700 | 222.4 | 234.3 | 236.8 | 243.7 | 239.5 | 209.3 |
| 800 | 225.0 | 237.0 | 239.6 | 247.1 | 242.7 | 211.8 |
| 900 | 227.0 | 239.3 | 241.9 | 249.8 | 245.4 | 214.0 |
| 1000 | 228.9 | 241.3 | 244.0 | 252.5 | 247.7 | 215.7 |
| 1100 | 230.6 | 243.0 | 245.8 | | 249.8 | 217.3 |
| 1300 | 233.3 | 246.0 | 248.8 | | 253.2 | 219.9 |
| 1600 | 236.6 | 249.5 | 252.4 | | 257.3 | 223.0 |
| 1800 | 238.4 | 251.4 | 254.3 | | 259.5 | 224.7 |
| 2000 | 240.1 | 253.1 | 256.1 | | 261.4 | 226.2 |

## MAGNESIUM PROPELLANT $\qquad P_o = 500$ PSI

| % Mg | SHIFTING $I_{SP}$ 500/14.7 | FROZEN $I_{SP}$ 500/14.7 | $T_o$ | $C_{P_g}$ (GAS) | R (GAS) | $\gamma_g$ (GAS) | WT % MgO |
|---|---|---|---|---|---|---|---|
| 0% | 227.0 | 224.6 | 4932 | 345.4 | 64.38 | 1.229 | 0% |
| 5% | 231.0 | 226.5 | 5119 | 345.9 | 67.08 | 1.242 | 8.29 % |
| 10% | 235.3 | 229.5 | 5345 | 353.1 | 70.72 | 1.251 | 16.58 % |
| 15% | 238.7 | 230.8 | 5491 | 358.9 | 74.38 | 1.262 | 24.87 % |
| 20% | 240.6 | 221.9 | 5531 | 362.1 | 77.66 | 1.274 | 33.16 % |

$C_s = 323.9 \frac{\text{FT-LBF}}{\text{LBM °R}}$ MgO @ 1000 °K

EFFICIENCIES

| % Mg | $\eta_{C.L.}$ | $\eta_{DIV}$ | $\eta_{KIN}$ | $\eta_{TP}$ | $\eta_{TOTAL}$ | | $I_{SP}$ DEL | P[illegible] |
|---|---|---|---|---|---|---|---|---|
| 0% | .9788 | .9830 | .9986 | 1.000 | .9608 | | 218.1 | 13.0 |
| 5% | .9788 | .9830 | .9974 | .9824 | .9428 | $\frac{7392.80}{7525.17}$ | 217.78 | 12.9 |
| 10% | .9788 | .9830 | .1167 | .9600 | .9206 | $\frac{7534.34}{7848.21}$ | 216.62 | 12.9 |
| 15% | .9788 | .9830 | .9956 | .9304 | .8913 | $\frac{7548.89}{8113.76}$ | 212.74 | 12.5 |
| 20% | .9788 | .9830 | .9996 | .8946 | .8518 | $\frac{7405.22}{8277.32}$ | 204.95 | 12.0 |

$$V_{eg} = \sqrt{2 c_P T_o \left[1 - \left(P_e/P_o\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$V_e = \sqrt{2\left[x C_s + (1-x) C_P\right] T_o \left[1 - \left(P_e/P_o\right)^{N}\right]}$$

$$N = \frac{R}{\left[x/(1-x)\right] C_s + C_P}$$

MAGNESIUM PROPELLANT $I_{SP}$
VARIATION WITH METAL CONTENT

84% TOTAL SOLIDS
(16% BINDER)

FROZEN FLOW

| % Mg | $P_c$ = 500 | $P_c$ = 1000 | $P_c$ = 2000 |
|---|---|---|---|
| 0% | 224.6 | 238.9 | 250.6 |
| 5% | 226.5 | 241.3 | 253.6 |
| 10% | 229.5 | 245.2 | 258.2 |
| 15% | 230.8 | 241.8 | 263.1 |
| 20% | 221.9 | 241.7 | 257.3 |
| 25% | 227.0 (?) | 234.9 | 262.7 |

$\frac{LB_f - SEC}{LB_m}$

SHIFTING EQUILIBRIUM (MAX. THEORETICAL)

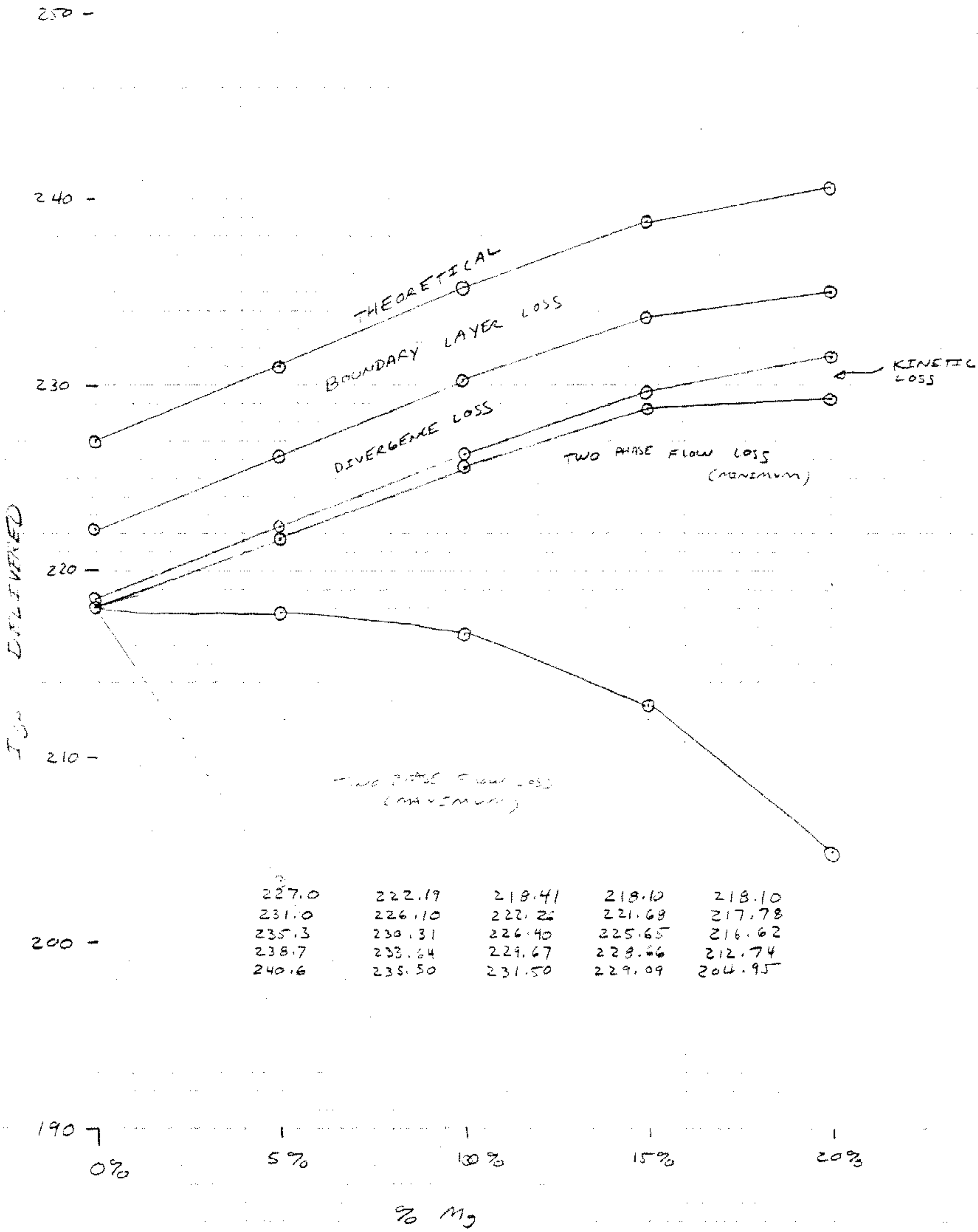
| % Mg | $P_c$ = 500 | $P_c$ = 1000 | $P_c$ = 2000 |
|---|---|---|---|
| 0% | 227.0 | 241.3 | 253.1 |
| 5% | 231.00 | 246.1 | 258.5 |
| 10% | 235.28 | 251.0 | 263.9 |
| 15% | 238.67 | 255.1 | 268.7 |
| 20% | 240.60 | 258.0 | 272.3 |
| 25% | 238.7 | 257.1 | 272.4 |

$\frac{LB_f - SEC}{LB_m}$

| AP | Mg | R-45 |
|---|---|---|
| 84% | 0% | 16% |
| 79% | 5% | 16% |
| 74% | 10% | 16% |
| 69% | 15% | 16% |
| 64% | 20% | 16% |
| 59% | 25% | 16% |

APPENDIX C

MAGNESIUM PROPELLANT
LOSSES $P_c = 500/14.7$




| | | | | |
|---|---|---|---|---|
| 227.0 | 222.19 | 218.41 | 218.10 | 218.10 |
| 231.0 | 226.10 | 222.22 | 221.68 | 217.78 |
| 235.3 | 230.31 | 226.40 | 225.65 | 216.62 |
| 238.7 | 233.64 | 229.67 | 228.66 | 212.74 |
| 240.6 | 235.50 | 231.50 | 229.09 | 204.95 |

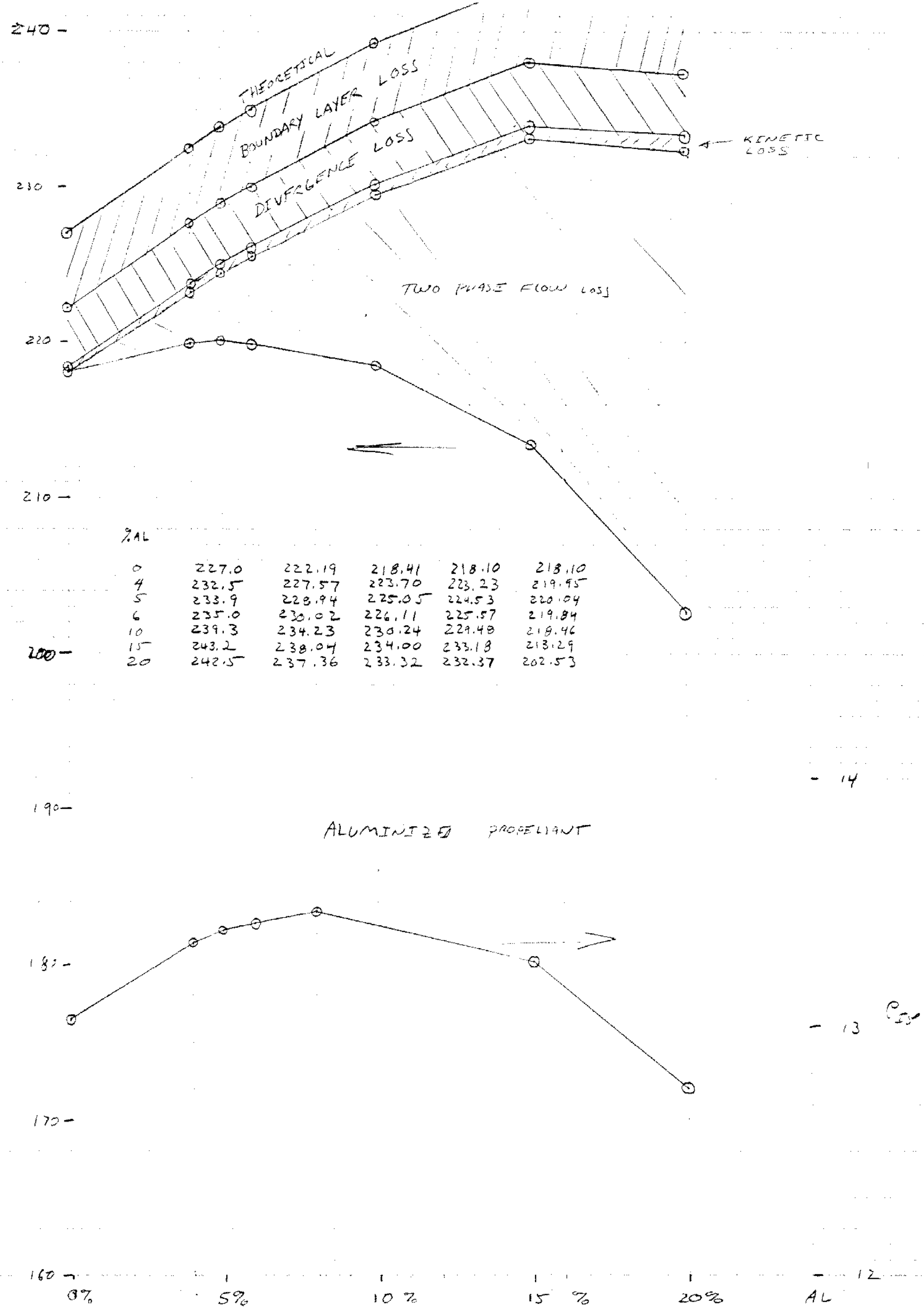
## ALUMINIZED PROPELLANT $P_0 = 500$ PSI $C_S = 808.5$

| % AL | SHIFTING $I_{SP}$ 500/14.7 | FROZEN $I_{SP}$ 500/14.7 | $T_0$ | $C_P$ (GAS) FT·LB/SLUG·°R | R (GAS) | $\gamma$ (GAS) | WT% $Al_2O_3$ |
|---|---|---|---|---|---|---|---|
| 0% | 227.0 | 224.6 | 4932 °R | 345.4 | 64.38 | 1.229 | 0% |
| 4% | 232.5 | 228.8 | 5158 °R | 353.5 | 68.06 | 1.233 | 7.558% |
| 5% | 233.9 | 229.9 | 5214 °R | 355.8 | 69.09 | 1.241 | 9.448% |
| 6% | 235.0 | 230.8 | 5266 °R | 353.0 | 70.15 | 1.244 | 11.337% |
| 10% | 239.3 | 234.1 | 5468 °R | 367.5 | 74.77 | 1.255 | 18.895% |
| 15% | 243.2 | 236.9 | 5674 °R | 379.7 | 81.12 | 1.272 | 28.343% |
| 20% | 242.5 | 235.1 | 5656 °R | 379.7 | 83.93 | 1.283 | 37.791% |

$C_{S_{Al_2O_3}} = 808.5$ (2000 °K)

## EFFICIENCIES

| % AL | $\eta_{B.L.}$ | $\eta_{DIV}$ | $\eta_{KIN}$ | $\eta_{TP}$ | | $I_{SP\ DEL}$ | $C_{ISP}$ |
|---|---|---|---|---|---|---|---|
| 0% | .9788 | .9830 | .9986 | 1.000 | | 218.1 | 13.03 |
| 4% | .9788 | .9830 | .9979 | .9853 | $\frac{7488.94}{7600.62}$ | 219.95 | 13.27 |
| 5% | .9788 | .9830 | .9977 | .9800 | $\frac{7539.78}{7693.66}$ | 220.04 | 13.31 |
| 6% | .9788 | .9830 | .9976 | .9746 | $\frac{7595.02}{[illegible]}$ | 219.34 | 13.33 |
| 10% | .9788 | .9830 | .9971 | .9517 | $\frac{7746.75}{8133.36}$ | [illegible] | 13.37 |
| 15% | .9788 | .9830 | .9965 | .9147 | $\frac{7337.88}{[illegible]}$ | 213.29 | 13.21 |
| 20% | .9788 | .9830 | .9959 | .8716 | $\frac{7563.83}{[illegible]}$ | [illegible] | 12.70 |




| %AL | | | | | |
|---|---|---|---|---|---|
| 0 | 227.0 | 222.19 | 218.41 | 218.10 | 218.10 |
| 4 | 232.5 | 227.57 | 223.70 | 223.23 | 219.95 |
| 5 | 233.9 | 228.94 | 225.05 | 224.53 | 220.04 |
| 6 | 235.0 | 230.02 | 226.11 | 225.57 | 219.84 |
| 10 | 239.3 | 234.23 | 230.24 | 229.48 | 219.46 |
| 15 | 243.2 | 238.04 | 234.00 | 233.18 | 213.29 |
| 20 | 242.5 | 237.36 | 233.32 | 232.37 | 202.53 |